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The following well-known relation

$$\int d\varphi\sqrt{\varphi^2+n^2} = \tfrac{1}{2}\varphi\sqrt{\varphi^2+n^2} + \tfrac{1}{2}n^2\log\left(\varphi+\sqrt{\varphi^2+n^2}\right)$$

facilitates the reduction of the preceding integral to

$$A_2 = 2\pi a\left[a + \frac{b^2}{\sqrt{a^2-b^2}}\log\left(\frac{a+\sqrt{a^2-b^2}}{b}\right)\right],$$

or

$$A_2 = 2\pi a^2\left[1 + \frac{1}{2}e^{-1}(1-e^2)\log\left(\frac{1+e}{1-e}\right)\right]. \tag{5}$$

Remarks. Both of the proposed expressions are incorrect. When $e = 1$ $(b = 0)$ the given formula for the prolate spheroid leads to $2\pi a^2$ instead of zero. Again, when $e = 0$ $(b = a)$ the series

$$\frac{1}{2}e^{-1}\log\left(\frac{1+e}{1-e}\right) = 1 + \frac{1}{3}e^2 + \frac{1}{5}e^4 + \cdots$$

shows that the given expression for the oblate spheroid reduces to $8\pi a^2$ instead of $4\pi a^2$ (sphere). Formulas (4) and (5) are equivalent to those given in Williamson's *Integral Calculus*, page 258.

Note. Contributors named below observed that the results as corrected are given in Lamb's *Infinitesimal Calculus*, 1902, page 273 and in Czuber's *Integralrechnung*, page 284. It is also given in *The Encyclopaedia Britannica*, ninth edition, volume 13, page 55.—Editors.

Also solved by Norman Anning, J. A. Bullard, Nathan Deutschman, A. Fayder, L. G. Feman, Maurice Kraut, Benjamin Levine, I. H. Marantz, Moses Nissenbaum, H. L. Olson, Arthur Pelletier, H. W. Reddick, H. A. Rhodes, J. L. Riley, D. H. Richert, J. B. Reynolds, and T. R. Thomson.

2815 [1920, 134]. Proposed by the late L. G. WELD.

A right circular cone is laid upon an inclined plane so that its element of contact makes a given angle with the slant line of the plane. Assuming that there is no slipping and that the rolling friction is negligible, find the time of oscillation of the cone.

I. Solution by J. B. Reynolds, Lehigh University.

The center of gravity of the cone moves in a circle of radius $\frac{3}{4}h\cos\alpha$ at a perpendicular distance $\frac{3}{4}h\sin\alpha$ from the plane, h being the height of the cone and 2α its vertical angle. When the radius of this circle makes an angle θ with the line of slope of the plane, the center of gravity of the cone has been raised a height $\frac{3}{4}h\cos\alpha(1-\cos\theta)\sin\beta$ above its lowest position, β being the angle of slope of the plane. Then the potential energy, *P.E.*, is $\frac{3}{4}mgh\cos\alpha\sin\beta(1-\cos\theta)$, m being the mass of the cone.

Since the line of contact is the instantaneous axis of rotation, the kinetic energy, *K.E.*, of the cone is $\frac{1}{2}I\dot{\varphi}^2$, I being the moment of inertia with respect to the instantaneous axis or an element of the cone and $\dot{\varphi}$ the angular velocity about this axis.

Now the moment of inertia of a thin disc of radius a and thickness t with respect to a line through its center making an angle α with a perpendicular to its plane is

$$t\int_0^{2\pi}\int_0^a r^2(\cos^2\theta\cos^2\alpha + \sin^2\theta)rdrd\theta = \frac{\pi a^4}{4}(1+\cos^2\alpha)t$$

and for the moment of inertia of the disc about a parallel line at a distance $a\cos\alpha$, we have

$$\frac{\pi a^4}{4}(1+\cos^2\alpha)t + \pi a^2t(a\cos\alpha)^2 = \frac{\pi a^4}{4}(1+5\cos^2\alpha)t.$$

So that, since the radius of any circular differential disc at distance x from the vertex of the cone is $x \tan \alpha$, we have

$$I = \int_0^h \frac{\pi}{4}(1 + 5\cos^2\alpha)\tan^4\alpha\, x^4 dx = \frac{\pi}{20}h^5(1 + 5\cos^2\alpha)\tan^4\alpha$$

$$= \tfrac{3}{20}mh^2\tan^2\alpha(1 + 5\cos^2\alpha),$$

since

$$m = \tfrac{1}{3}\pi h^3\tan^2\alpha.$$

By the principle of the conservation of energy

$$K.E. + P.E. = \text{constant}$$

or

$$\tfrac{1}{2}\tfrac{3}{20}mh^2\tan^2\alpha(1 + 5\cos^2\alpha)\dot{\varphi}^2 + \tfrac{3}{4}mgh\cos\alpha\sin\beta(1 - \cos\theta) = \text{constant};$$

or since $\dot{\varphi} = \dot{\theta}\cot\alpha$, we get

$$\tfrac{1}{10}h^2(1 + 5\cos^2\alpha)\dot{\theta}^2 + gh\cos\alpha\sin\beta(1 - \cos\theta) = \text{constant}.$$

Differentiating this equation with respect to t and cancelling out $\dot{\theta}$ we get,

$$\ddot{\theta} = -\frac{5g\cos\alpha\sin\beta}{h(1 + 5\cos^2\alpha)}\sin\theta,$$

from which we have, if θ is small, for the time T for an oscillation

$$T = 2\pi\sqrt{\frac{h(1 + 5\cos^2\alpha)}{5g\cos\alpha\sin\beta}}.$$

If the oscillations are not small and γ is the given angle that the element of contact initially makes with the line of greatest slope

$$T = 2\pi\sqrt{\frac{g(1 + 5\cos^2\alpha)}{5g\cos\alpha\sin\beta}}\left\{1 + \frac{1}{4}\sin^2\frac{\gamma}{2} + \frac{9}{64}\sin^4\frac{\gamma}{2} + \cdots + \left[\frac{n\,\underline{|2n-1}}{2^{2n-1}(\underline{|n})^2}\right]^2\sin^{2n}\frac{\gamma}{2} + \cdots\right\}.$$

II. SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Assume the vertex of the cone as fixed in the inclined plane; then we have the motion of a solid of revolution with one point fixed; and the motion is represented by Euler's equations, employing the usual notation, in the form

$$\begin{aligned} A\dot{\omega}_1 + (C - A)\omega_2\omega_3 &= L, \\ A\dot{\omega}_2 - (C - A)\omega_3\omega_1 &= M, \\ C\dot{\omega}_3 &= N, \end{aligned} \tag{1}$$

the two equal moments of inertia about axes through the vertex of the cone and perpendicular to the axis of the cone and to each other being given by A, and the moment of inertia about the axis of the cone by C.

Also take the line of contact of the cone and plane as an instantaneous axis, and let its direction angles be λ, μ, ν, and the vertical angle of the cone $= 2\alpha$, $\varphi =$ the angle through which the cone rotates after any time t from the beginning of motion; so that, the x-axis being initially in the inclined plane, φ also gives the rotary motion of the x-axis; and $\omega =$ the instantaneous angular velocity about the instantaneous axis.

Let $\rho =$ the distance of any point in the instantaneous axis from the origin; then as in solid analytic geometry,

$$\cos\lambda = \frac{x}{\rho} = \sin\alpha\sin\varphi, \qquad \cos\mu = \frac{y}{\rho} = \sin\alpha\cos\varphi, \qquad \cos\nu = \cos\alpha; \tag{2}$$

and, by the theory of rotary motion,

$$\omega_1 = \omega\sin\alpha\sin\varphi, \qquad \omega_2 = \omega\sin\alpha\cos\varphi, \qquad \omega_3 = \omega\cos\alpha. \tag{3}$$

We have

$$\omega = \dot{\varphi}. \tag{4}$$

If $m =$ the mass of the cone, and $h =$ its altitude,

$$A = \tfrac{3}{20}mh^2(4 + \tan^2\alpha), \qquad C = \tfrac{3}{10}mh^2\tan^2\alpha. \tag{5}$$

Let $\dot\theta$ = the angular motion of the instantaneous axis on the plane; this is the angular motion about the line (n) which is normal to the inclined plane and passes through the vertex of the cone. A plane perpendicular to the inclined plane includes this normal, the instantaneous axis (a_i), the axis of the cone, and a perpendicular (p) to the axis of the cone. The angle between n and p is α and between a_i and p is $\pi/2 + \alpha$. The components of $\dot\theta$ and ω about p are $\dot\theta \cos\alpha$ and $\omega \cos(\pi/2 + \alpha) = -\omega \sin\alpha$, but these denote the same motion; then

$$\dot\theta \cos\alpha = -\omega \sin\alpha \qquad \text{or} \qquad \omega = -\dot\theta \cot\alpha. \tag{6}$$

Now the moment of the external forces about the instantaneous axis

$$= L\cos\lambda + M\cos\mu + N\cos\nu; \tag{7}$$

also

$$= \tfrac{3}{4} mgh \sin\alpha \sin\beta \sin\theta \tag{8}$$

β being the inclination of the plane.

Substituting (4) in (3) and differentiating, we have

$$\dot\omega_1 = \sin\alpha(\cos\varphi\cdot\dot\varphi^2 + \sin\varphi\cdot\ddot\varphi), \quad \dot\omega_2 = \sin\alpha(-\sin\varphi\cdot\dot\varphi^2 + \cos\varphi\cdot\ddot\varphi), \quad \text{and} \quad \dot\omega_3 = \cos\alpha\cdot\ddot\varphi, \tag{9}$$

Now put the values given by (3), (5), (9) in (1), and we have the values of L, M, N; and using these, with (2), (7) and (8), we have the single equation of motion

$$\frac{3m}{20}(6 + \tan^2\alpha)h^2 \sin^2\alpha\cdot\ddot\varphi = \frac{3m}{4} gh \sin\alpha \sin\beta \sin\theta. \tag{10}$$

Now from (4) and (6),

$$\ddot\theta = -\frac{g}{h}\frac{5 \sin\beta \cos\alpha}{1 + 5\cos^2\alpha} \sin\theta. \tag{11}$$

If θ be so small that θ may displace $\sin\theta$, we have the required time of oscillation

$$T = 2\pi\sqrt{\frac{1 + 5\cos^2\alpha}{5 \sin\beta \cos\alpha}\cdot\frac{h}{g}}.$$

Also solved by F. L. WILMER, and discussed by C. H. ECKART.

2820 [1920, 134]. Proposed by C. B. HALDEMAN, Ross, Ohio.

Given one angle and the radii of the inscribed and circumscribed circles, to construct the triangle geometrically.

SOLUTION BY A. V. RICHARDSON, Bishop's College, Lennoxville, Quebec.

With the usual notation, A, R, r are given. Draw a circle of radius R, and let KB be any diameter. (The reader is requested to draw the figure). Make the angle, $BKC = A$. Draw XY parallel to BC and at a distance r from it on the same side as K. Bisect the arc BC at V, and let the circle, center V, radius VC, cut XY at I and I'.

These points will be the incenters for the two (symmetrical) solutions. Let VI meet the circle again in A. Then ABC is the required triangle. To show this, it is only necessary to prove that the straight line CI bisects the angle C. Since $VI = VC$, $\angle VIC = \angle VCI$. But $\angle VIC = (A/2) + \angle ICA$ and $\angle VCI = (A/2) + \angle BCI$. Hence $\angle ICA = \angle BCI$ and I is the intersection of two bisectors of the angles of the triangle ABC.

Also solved by L. C. MATHEWSON, H. L. OLSON, ARTHUR PELLETIER, J. B. REYNOLDS, JOSEPH ROSENBAUM, C. N. SCHMALL, and the Proposer.

2829 [1920, 226]. Proposed by E. S. PALMER, New Haven, Conn.

Given a set of arbitrary pairs of positive integers (a_p, b_p), $(p = 1, 2, \cdots, n)$: (a) Is it always possible to find a set of positive integers k_p, $(p = 1, 2, \cdots, n)$ such that

$$k_p a_p + k_p b_p > \sum_{r=1}^{r=n} k_r a_r, \qquad (p = 1, 2, 3, \cdots, n).$$

(b) If or when possible, show how to find k_p.